MODEL ANSWERS

Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
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**Pearson Edexcel Level 3 GCE**

Centre Number | Candidate Number

# Thursday 14 May 2020

Afternoon | Paper Reference **8FM0/26**

# Further Mathematics

## Advanced Subsidiary
## Further Mathematics options
## 26: Further Mechanics 2
## (Part of option J)

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of $g$ is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 3 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

**1.**

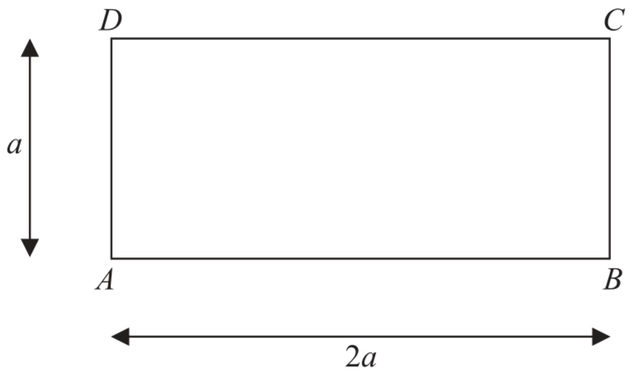



**Figure 1**

Figure 1 shows a uniform rectangular lamina $ABCD$ with $AB = 2a$ and $AD = a$
The mass of the lamina is $6m$.

A particle of mass $2m$ is attached to the lamina at $A$, a particle of mass $m$ is attached to the lamina at $B$ and a particle of mass $3m$ is attached to the lamina at $D$, to form a loaded lamina $L$ of total mass $12m$.

(a) Write down the distance of the centre of mass of $L$ from $AB$. You must give a reason for your answer.

**(2)**

(b) Show that the distance of the centre of mass of $L$ from $AD$ is $\frac{2a}{3}$

**(3)**

A particle of mass $km$ is now also attached to $L$ at $D$ to form a new loaded lamina $N$.

(c) Show that the distance of the centre of mass of $N$ from $AB$ is $\frac{(k+6)a}{(k+12)}$

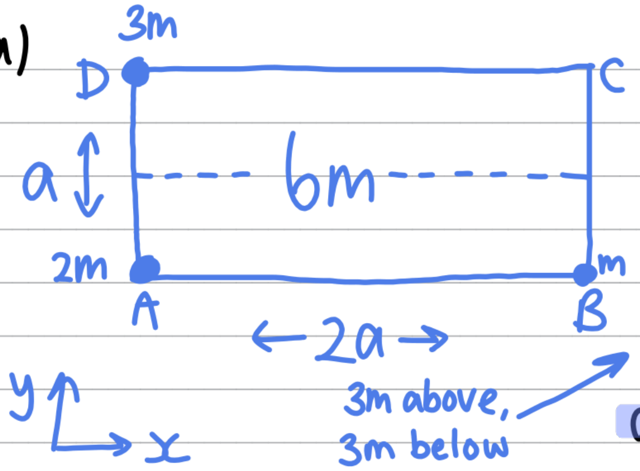
**(4)**

When $N$ is freely suspended from $A$ and is hanging in equilibrium, the side $AB$ makes an angle $\alpha$ with the vertical, where $\tan\alpha = \frac{3}{2}$

(d) Find the value of $k$.

**(6)**

a)




centre of mass, G, is $\frac{a}{2}$ from AB $\because$ distribution of mass on lamina is symmetrical about perpendicular bisector of AD

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**Question 1 continued**

b) $\sum m_i x_i = \bar{x} \sum m_i$

moments about AD: pick axes & origin

rectangle is uniform so C.O.M. is @ its centre, $(a, \frac{a}{2})$

$$6m\begin{pmatrix} a \\ \frac{a}{2} \end{pmatrix} + 3m\begin{pmatrix} 0 \\ a \end{pmatrix} + 2m\begin{pmatrix} 0 \\ 0 \end{pmatrix} + m\begin{pmatrix} 2a \\ 0 \end{pmatrix} = 12m\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

we only need $\bar{x}$: $6ma + 2ma = 12m\bar{x}$

$$\Rightarrow \boxed{\bar{x} = \frac{2a}{3}}$$

c) $\sum m_i y_i = \bar{y} \sum m_i$

moments about AB:

$$\underbrace{6m\begin{pmatrix} a \\ \frac{a}{2} \end{pmatrix} + 3m\begin{pmatrix} 0 \\ a \end{pmatrix} + 2m\begin{pmatrix} 0 \\ 0 \end{pmatrix} + m\begin{pmatrix} 2a \\ 0 \end{pmatrix}} + km\begin{pmatrix} 0 \\ a \end{pmatrix} = (k+12)m\bar{y}$$

from (a), know $\bar{y}$ for L is $\frac{a}{2}$, so $\bar{y}$ for N is given by

$12\cancel{m} \cdot \frac{a}{2} + k\cancel{m}a = (k+12)\cancel{m}\bar{y} \Rightarrow 6a + ka = (k+12)\bar{y}$

$$\Rightarrow \boxed{\bar{y} = \frac{(k+6)a}{k+12}}$$

**Question 1 continued**

d) when freely suspended, C.O.M. will be directly below point of suspension.

so we need $\bar{x}$ for N:

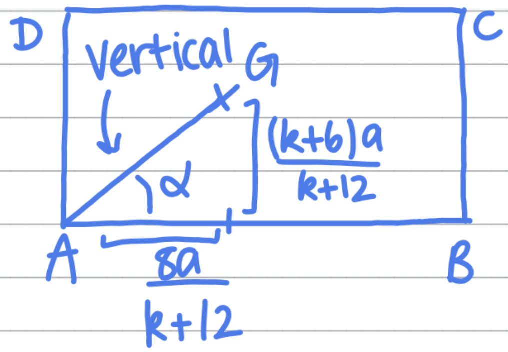
Moments about AD

$$12\cancel{m}\left(\frac{2a}{3}\right) = (12+k)\cancel{m}\,\bar{x}$$

(b) →

$$8a = (12+k)\,\bar{x}$$

$$\bar{x} = \frac{8a}{k+12}$$




$$\tan\alpha = \frac{\bar{y}}{\bar{x}} = \frac{\frac{(k+6)\cancel{a}}{\cancel{(k+12)}}}{\frac{8\cancel{a}}{\cancel{(k+12)}}} = \frac{k+6}{8}$$

if $\frac{k+6}{8} = \frac{3}{2}$, $\Rightarrow \frac{k+6}{8} = \frac{12}{8} \Rightarrow \boxed{k=6}$

**Question 1 continued**

**(Total for Question 1 is 15 marks)**

**2.**

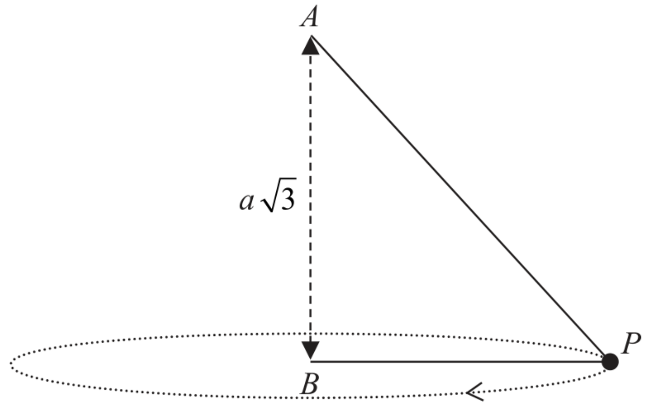



**Figure 2**

One end of a string of length $3a$ is attached to a point $A$ and the other end is attached to a point $B$ on a smooth horizontal table. The point $B$ is vertically below $A$ with $AB = a\sqrt{3}$ A small smooth bead, $P$, of mass $m$ is threaded on to the string. The bead $P$ moves on the table in a horizontal circle, with centre $B$, with constant speed $U$. Both portions, $AP$ and $BP$, of the string are taut, as shown in Figure 2.

The string is modelled as being light and inextensible and the bead is modelled as a particle.

(a) Show that $AP = 2a$ **(2)**

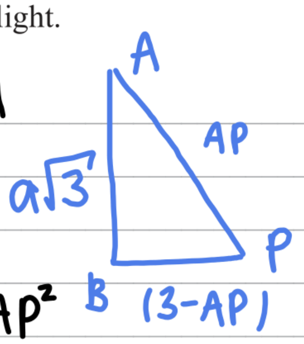
(b) Find, in terms of $m$, $U$ and $a$, the tension in the string. **(4)**

(c) Show that $U^2 < ag\sqrt{3}$ **(5)**

(d) Describe what would happen if $U^2 > ag\sqrt{3}$ **(1)**

(e) State briefly how the tension in the string would be affected if the string were not modelled as being light. **(1)**

a) $APB = 3a$




Pythagoras:

$(a\sqrt{3})^2 + (3a - AP)^2 = AP^2$

$3a^2 + 9a^2 - 6aAP + AP^2 = AP^2$

$12a^2 = 6aAP$

$\therefore$ $AP = 2a$

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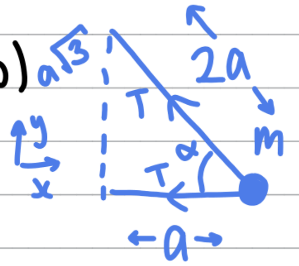
**Question 2 continued**

b) 


centripetal force is horizontal $\Rightarrow$ resolve into

$x$ components: $T + T\cos\alpha = \frac{mv^2}{r}$

tension in string is constant

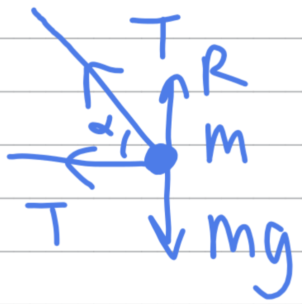
$\cos\alpha = \frac{a}{2a} = \frac{1}{2} \Rightarrow T + \frac{1}{2}T = \frac{mU^2}{a}$

$\frac{3}{2}T = \frac{mU^2}{a}$

$T = \frac{2mU^2}{3a}$

c) presence of 'g' in expression hints we need to consider weight

resolve into y components:




$(\uparrow)\ R + T\sin\alpha = W\ (\downarrow)$

$\sin\alpha = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2} \Rightarrow R + \frac{\sqrt{3}}{2}T = mg$

$\Rightarrow R + \frac{\sqrt{3}}{2}\left(\frac{2mU^2}{3a}\right) = mg$

for bead to remain on table, $R > 0$

$\therefore mg - \frac{2mU^2\sqrt{3}}{3a \times 2} > 0$

$g - \frac{U^2}{\sqrt{3}a} > 0$

$\therefore U^2 < ag\sqrt{3}$

**Question 2 continued**

d) if $U^2 > ag\sqrt{3}$, $R < 0$ so bead would lift off the table

e) then weight of string contributes to balance of forces $\Rightarrow$

tension varies along string

**Question 2 continued**

**(Total for Question 2 is 13 marks)**

**3.** At time $t = 0$, a toy electric car is at rest at a fixed point $O$. The car then moves in a horizontal straight line so that at time $t$ seconds ($t > 0$) after leaving $O$, the velocity of the car is $v\,\mathrm{m\,s^{-1}}$ and the acceleration of the car is modelled as $(p + qv)\,\mathrm{m\,s^{-2}}$, where $p$ and $q$ are constants.

When $t = 0$, the acceleration of the car is $3\,\mathrm{m\,s^{-2}}$

When $t = T$, the acceleration of the car is $\frac{1}{2}\,\mathrm{m\,s^{-2}}$ and $v = 4$

(a) Show that

$$8\frac{dv}{dt} = (24 - 5v)$$

**(6)**

(b) Find the exact value of $T$, simplifying your answer.

**(6)**

a) initial conditions: $t=0, v=0, a=3$

$a = p + qv \Rightarrow a_0 = 3 = p \Rightarrow p = 3$

@ $t = T, a = \frac{1}{2}$ & $v = 4 \Rightarrow a_T = p + qv = 3 + 4q = \frac{1}{2}$

$4q = -\frac{5}{2}$

$q = -\frac{5}{8}$

$a = \frac{dv}{dt} = 3 - \frac{5}{8}v$

$\times 8$: $8\frac{dv}{dt} = 24 - 5v$ (as required)

b) we have $\frac{dv}{dt}$, so can integrate from $v = 0$ to $v = 4$ over time interval $T$

$8\frac{dv}{dt} = 24 - 5v \Rightarrow \frac{8\,dv}{24 - 5v} = dt$

**Question 3 continued**

$$8\int \frac{dv}{(24-5v)} = \int dt$$

$$8\left(-\frac{1}{5}\ln|24-5v|\right) = t + C$$

initial conditions: $-\frac{8}{5}\ln(24-5(0)) = (0) + C$

so $C = -\frac{8}{5}\ln 24$

when $v = 4$

@ $t = T$: $-\frac{8}{5}\ln(24-5(4)) = T - \frac{8}{5}\ln 24$

$$-\frac{8}{5}\ln(4) = T - \frac{8}{5}\ln 24$$

$$T = \frac{8}{5}\ln\left(\frac{24}{4}\right)$$

$$\Rightarrow T = \frac{8}{5}\ln 6$$

**Question 3 continued**

**(Total for Question 3 is 12 marks)**

**TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS**